THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH2050B Mathematical Analysis I (Fall 2016)
Tutorial Questions for 27 Oct

1. Show by definition of limits of functions that

   (a)$\lim_{x\to 3} \dfrac{2x+3}{4x-9} = 3$

   (b)$\lim_{x\to 1} \dfrac{x^3-1}{x^2-3x+2} = -3$

2. Let $f : A \subseteq \mathbb{R}, f : A \to \mathbb{R}$, and $c \in \mathbb{R}$ be a cluster point of A. Then we have

   Theorem 1. *(a) (Sequential Criterion, version I)* $\lim_{x\to c} f(x) = l \in \mathbb{R}$ *if and only if for each sequence* $\{x_n\} \subseteq A\backslash\{c\}$ *such that* $\lim_{n\to\infty} x_n = c$*, we have* $\lim_{n\to\infty} f(x_n) = l$.

   (b) (Sequential Criterion, version II) $\lim_{x\to c} f(x)$ *exists in* $\mathbb{R}$ *if and only if for each sequence* $\{x_n\} \subseteq A\backslash\{c\}$ *such that* $\lim_{n\to\infty} x_n = c$*, we have* $\lim_{n\to\infty} f(x_n)$ *exists in* $\mathbb{R}$.

3. Let $f : A \subseteq \mathbb{R}, f : A \to \mathbb{R}$, and $c \in \mathbb{R}$ be a cluster point of A. Then we have

   Theorem 2. *(Divergence Criterioa)*

   (a) $f(x)$ *does not have the limit* $l \in \mathbb{R}$ *at* c *if and only if there is a sequence* $\{x_n\} \subseteq A\backslash\{c\}$ *such that* $\lim_{n\to\infty} x_n = c$*, but* $f(x_n)$ *does not converge to* l.

   (b) $\lim_{x\to c} f(x)$ *does not exist in* $\mathbb{R}$ *if and only if there is a sequence* $\{x_n\} \subseteq A\backslash\{c\}$ *such that* $\lim_{n\to\infty} x_n = c$*, but* $\lim_{n\to\infty} f(x_n)$ *does not exist in* $\mathbb{R}$.

   (c) $\lim_{x\to c} f(x)$ *does not exist if we can find two sequences* $\{x_n\}$, $\{y_n\} \subseteq A\backslash\{c\}$ *such that* $\lim_{n\to\infty} x_n = c$*, with* $f(x_n) \to x$, $f(y_n) \to y$*, but* $x \neq y$.

4. Using the definition or any divergence criteria, show that the following limits do not exist.

   (a) $\lim\limits_{x\to 0} \sin(\dfrac{1}{x})$

   (b) $\lim\limits_{x\to 1} \dfrac{2}{1-x}$

   (c) $\lim\limits_{x\to 0} \dfrac{|x|}{x}$